Algebraic Geometry I Prof. Dr. Ulrich Görtz
WS 2022/23 Dr. Heer Zhao

Problem sheet 8

Due date: Dec. 06, 2022.

Problem 29

Let X be a topological space, let Z be a subset of X (equipped with the subspace topology) and let $\iota\colon Z \to X$ be the inclusion map. Let $\mathscr{F}$ be a sheaf on Z.

(1) Let $\overline{Z}$ denote the closure of Z in X, and let $x \in X \setminus \overline{Z}$. Show that the stalk $(\iota_*\mathscr{F})_x$ is a singleton set.

(2) Let $x \in Z$. Show that there is a natural isomorphism $(\iota_*\mathscr{F})_x \cong \mathscr{F}_x$.

Problem 30

Let X be a topological space, $U \subseteq X$ open, and denote by $j\colon U \to X$ the inclusion map. Let $\mathscr{F}$ be a sheaf of abelian groups on U. Denote by $j_!(\mathscr{F})$ the sheaf associated with the following presheaf on X:

$$V \mapsto \begin{cases} \mathscr{F}(V) & \text{if } V \subseteq U, \\ 0 & \text{otherwise,} \end{cases} \qquad \text{for } V \subseteq X \text{ open}$$

Compute the stalks of $j_!(\mathscr{F})$ and the restriction $j_!(\mathscr{F})_{|U}$. It is easy to define $j_!$ on sheaf morphisms, so that $j_!$ is a functor. Find a functor which is right adjoint to $j_!$.

Problem 31

Give an example of affine schemes X, Y and a morphism $X \to Y$ of ringed spaces which is not a morphism of locally ringed spaces.

Problem 32

Let A be a domain with field of fractions K. We view all (non-trivial) localizations of A as subrings of K. Let $f \in A$, $f \neq 0$. Show that

$$A_f = \bigcap_{\mathfrak{p} \in D(f)} A_{\mathfrak{p}}.$$

Hint. Given $g \in \bigcap_{\mathfrak{p} \in D(f)} A_{\mathfrak{p}}$, consider the ideal

$$\mathfrak{a} = \{h \in A;\ hg \in A\} \subseteq A.$$