Problem sheet 13

Due date: Jan. 24, 2022.

Problem 49

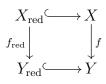
- (1) Let X be an irreducible noetherian scheme with generic point η such that the local ring $\mathscr{O}_{X,\eta}$ is reduced. Show that there exists a non-empty open subscheme $U \subseteq X$ which is reduced.
- (2) Give an example of a non-reduced irreducible noetherian scheme X such that the local ring at the generic point is reduced.

Problem 50

Show that open and closed immersions are *monomorphisms* in the category of schemes: If $f: X \to Y$ is an open (closed) immersion, then for every scheme S the induced map $\operatorname{Hom}(S, X) \to \operatorname{Hom}(S, Y)$ is injective.

Problem 51

- (1) Let X be a scheme. Prove that there exists a unique reduced closed subscheme X_{red} of X which has the same underlying topological space as X.
- (2) Let $f: X \to Y$ be a morphism of schemes. Show that f induces a unique morphism $f_{\text{red}}: X_{\text{red}} \to Y_{\text{red}}$ such that the diagram



is commutative.

Problem 52

Let k be a field, and let $A = k[X, Y]/(XY, X^2)$. Define two morphisms

$$f, g: \operatorname{Spec} A \to \operatorname{Spec} k[T]/(T^2)$$

such that $f \neq g$, but such that there exists a non-empty open subset $U \subset \operatorname{Spec} A$ such that $f_{|U} = g_{|U}$.