Algebraic Geometry I Prof. Dr. Ulrich Görtz
WS 2018/19

Problem sheet 9

Due date: Dec. 11, 2018.

Problem 37

Let k be a field. Let X be the scheme obtained by gluing two copies of $\mathbb{A}^1_k = \operatorname{Spec} k[T]$ along the open subset $U = D(T)$, with respect to the identity map $U \to U$ ("the affine line with the origin doubled"). Prove that X is not an affine scheme.

Problem 38

Let k be an infinite field, let $n \geqslant 1$, and let $U \subseteq \mathbb{A}^n_k$ be a non-empty open subscheme. Show that there exists a morphism $\operatorname{Spec} k \to U$ of k-schemes (i.e., that U has a k-valued point).

Give an example of an infinite field k and a non-empty affine k-scheme X which has no k-valued points.

Problem 39

Let k be an algebraically closed field. Give an example of a morphism of schemes $\mathbb{A}^2_k \to \mathbb{P}^1_k$ which is surjective (i.e., the underlying map on topological spaces is surjective).

Hint. First look for a map $\mathbb{A}^2_k(k) = k^2 \to \mathbb{P}^1(k)$ which is "defined by polynomials", i.e., of the form $(x, y) \mapsto (f(x, y) \colon g(x, y))$ for polynomials $f, g \in k[X, Y]$, and which is surjective.

Problem 40

Let R be a ring, $n \geqslant 0$. Show that the natural homomorphism $R \to \Gamma(\mathbb{P}^n_R, \mathscr{O}_{\mathbb{P}^n_R})$ is an isomorphism.