Algebraic Geometry I Prof. Dr. Ulrich Görtz
WS 2018/19

Problem sheet 6

Due date: Nov. 20, 2018.

Problem 25

Let X be an irreducible topological space, E a set, and $\mathscr{F}$ the constant sheaf on X associated with E. Show that $\mathscr{F}(U) = E$ for every non-empty open set $U \subseteq X$.

Problem 26

Let X be a topological space, and let $\mathscr{F}$, $\mathscr{G}$ be sheaves of abelian groups on X. For every open $U \subseteq X$, denote by $\mathrm{Hom}(\mathscr{F}_{|U}, \mathscr{G}_{|U})$ the abelian group of morphisms $\mathscr{F}_{|U} \to \mathscr{G}_{|U}$ of sheaves of abelian groups on U. This defines a presheaf in a natural way. Show that this presheaf is a sheaf.

Problem 27

Let X be a topological space, $U \subseteq X$ open, and denote by $j\colon U \to X$ the inclusion map. Let $\mathscr{F}$ be a sheaf of abelian groups on U. Denote by $j_!(\mathscr{F})$ the sheaf associated with the following presheaf on X:

$$V \mapsto \begin{cases} \mathscr{F}(V) & \text{if } V \subseteq U, \\ 0 & \text{otherwise,} \end{cases} \qquad \text{for } V \subseteq X \text{ open}$$

Compute the stalks of $j_!(\mathscr{F})$ and the restriction $j_!(\mathscr{F})_{|U}$. It is easy to define $j_!$ on sheaf morphisms, so that $j_!$ is a functor. Find a functor which is right adjoint to $j_!$.

Problem 28

Let $(X, \mathscr{O}_X)$ be a locally ringed space which is not connected. Prove that there exists a non-trivial idempotent element in $\Gamma(X, \mathscr{O}_X)$. (Cf. Problem 9.)