Algebraic Geometry I Prof. Dr. Ulrich Görtz
WS 2018/19

Problem sheet 4

Due date: Nov. 6, 2018.

Problem 13

Let I be a directed partially ordered set. Let $J \subset I$ be a *cofinal* subset, i.e., for all $i \in I$ there exists $j \in J$ with $j \geqslant i$. We regard J as a partially ordered set by restricting the partial order on I. Let $(X_i)_{i\in I}$ be an inductive system indexed by I. We can then regard $(X_j)_{j\in J}$ as an inductive system indexed by J. Prove that there is a natural isomorphism

$$\operatorname*{colim}_{j\in J} X_j \xrightarrow{\sim} \operatorname*{colim}_{i\in I} X_i.$$

Problem 14

Let I be a directed partially ordered set, let $(X_i)_i$ and $(Y_i)_i$ be inductive systems of sets indexed by I, and let X and Y be the respective colimits. Let $(u_i \colon X_i \to Y_i)_i$ a morphism of inductive systems (i.e., a family of maps compatible with the transition maps), and let $u \colon X \to Y$ be the induced map.

1. Show that if there exists $i \in I$ such that u_j is injective for all $j \geqslant i$, then u is injective.
2. Show that if there exists $i \in I$ such that u_j is surjective for all $j \geqslant i$, then u is surjective.

Problem 15

Let X be a topological space and let $\mathscr{F}$ be a sheaf on X. Let $U \subseteq X$ be an open subset and $s, t \in \mathscr{F}(U)$. Show that the set of $x \in U$ with $s_x = t_x \in \mathscr{F}_x$ is open in X.

Problem 16

Give an example of a topological space X, a surjective map $\mathscr{F} \to \mathscr{G}$ of sheaves on X and an open $U \subseteq X$ such that the map $\mathscr{F}(U) \to \mathscr{G}(U)$ is not surjective.