# Problem sheet 12

Due date: Jan. 15, 2018.

### Problem 49

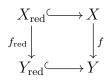
- (1) Show that the properties "open immersion" and "closed immersion" are *local* on the target: Given a morphism  $f: X \to Y$  and a cover  $Y = \bigcup_i V_i$  by open subschemes, f is an open (closed) immersion if and only if for every i, the induced morphism  $f^{-1}(V_i) \to V_i$  is an open (closed) immersion.
- (2) Show that the properties "open immersion" and "closed immersion" are stable under composition of morphisms.

#### Problem 50

Show that open and closed immersions are *monomorphisms* in the category of schemes: If  $f: X \to Y$  is an open (closed) immersion, then for every scheme S the induced map  $\operatorname{Hom}(S, X) \to \operatorname{Hom}(S, Y)$  is injective.

## Problem 51

- (1) Let X be a scheme. Prove that there exists a unique reduced closed subscheme  $X_{\text{red}}$  of X which has the same underlying topological space as X.
- (2) Let  $f: X \to Y$  be a morphism of schemes. Show that f induces a unique morphism  $f_{\text{red}}: X_{\text{red}} \to Y_{\text{red}}$  such that the diagram



## Problem 52

Let k be a field, and let  $A = k[X, Y]/(XY, X^2)$ . Define two morphisms  $f, g: \text{Spec } A \to \text{Spec } k[T]/(T^2)$  such that  $f \neq g$ , but such that there exists a non-empty open subset  $U \subset \text{Spec } A$  such that  $f_{|U} = g_{|U}$ .